Performance of Turbo-coded MC-CDMA System
Based on Complex Wavelet Packet in Rayleigh Fading Channel

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Abstract—In this paper, on the basis of analyzing the principle of multi-carrier code division multiple access (MC-CDMA) technique, by employing the optimized complex wavelet packet as multi-carrier modulation and the turbo codes as channel coding, we propose a novel MC-CDMA system based on complex wavelet packet and turbo coding, and investigate the system bit error rate (BER) performance in Rayleigh fading channel. The system can overcome the decrease of spectrum efficiency and energy of conventional MC-CDMA due to inserting cyclic prefix (CP); and make full use of the turbo codes' good capacity against fading channel to improve the BER performance further. Theoretical analysis and simulation results all show that the proposed system outperforms conventional MC-CDMA system, and its performance is superior to that of the conventional MC-CDMA with CP. Meanwhile, the application of turbo coding strengthens the system ability to cope with multi-path fading and multi-access interference (MAI) significantly.

Keywords: complex wavelet packet; turbo coding; multi-carrier technique; CDMA; OFDM

I. INTRODUCTION

Recently, Multi-carrier CDMA technique based on the combination of OFDM and conventional CDMA has received much attention among researchers, it is one of the most promising techniques for the future wireless mobile communication systems beyond 3G [1]. It has the properties desirable for high data-rate wireless services such as insensitivity to frequency selective channels, frequency diversity, high spectral efficiency and flexibility to generate different data rates within a fixed bandwidth [2]. However, the conventional MC-CDMA is implemented by means of IDFT and DFT operators. In its frequency spectrum, the main lobe doesn't concentrate energy effectively and side lobe attenuates slowly; the multi-path fading or synchronization error will cause severe performance degradation due to the inter-channel interference (ICI), ISI and MAI. To overcome the above shortcomings, a number of improved MC-CDMA systems have been proposed. Among them, wavelet packet based MC-CDMA system [3] has advantages of stronger ability to combat multi-path interference (MPI) and ISI than conventional DFT based MC-CDMA (DFT-MC-CDMA). Especially, optimized complex wavelet packet [4] not only has good properties that real wavelet packet possesses, such as shifting orthogonality, adaptability, time-frequency localization, etc., but also matches complex channel frequency spectrum and suits multi-carrier communications. Due to this, a optimized complex wavelet packet based MC-CDMA (CWP-MC-CDMA) system is presented in this paper, and corresponding downlink performance is investigated in Rayleigh fading channel, it can achieve better performance than conventional MC-CDMA system. Moreover, in fading channel, the channel coding technique can increase the communication system capacity effectively and improve the system BER performance, especially, turbo coding technique receives great interest because of its superior performance at lower SNR (signal to noise ratio), it can achieve near Shannon limits on the additive white Gaussian channel [5]. As a result of this dramatic evolution, turbo coding has also adopted by 3GPP as error correcting code for high-rate data service. For the reason above, we apply turbo coding to the CWP-MC-CDMA system, and develop a novel turbo coded MC-CDMA scheme based on complex wavelet packet. On one hand, by utilizing superior characteristics of optimized complex wavelet packet, we can overcome the decrease of spectrum efficiency and energy of conventional MC-CDMA system due to inserting to CP. On the other hand, we can make use of soft decision information from turbo codes' multiple iterations decoding to avoid performance loss due to hard decision, as well as random interleaver and good ability against the burst error of fading channel to perfect system performance further. Theoretical analysis and simulation results show the developed system outperforms conventional MC-CDMA, and has the superior capacity against the fading and MAI.

II. SYSTEM MODEL

The transmitter and receiver of turbo coded CWP-MC-CDMA system are illustrated in Fig.1 and 2, respectively. The DFT/IDFT of conventional MC-CDMA is replaced by CWPT (complex wavelet packet...
transform/inverse CWPT (ICWPT) accordingly. At the transmitter, the information bits \{b_k\} are first encoded by turbo coder, then the encoded bits \{x_k\} are mapped to QPSK constellation to form complex symbols \{\hat{a}_k\}. The output symbols are duplicated to different carriers, and the symbol over different carrier is multiplied by a chip of the spreading code. Then ICWPT is used to realize multi-carrier modulation. After modulation to RF, the turbo coder, then the encoded bits spreading code. Then ICWPT are sent into channel equalizer. After despreading and multi-carrier demodulation. After that, the output signals symbol over different carrier is multiplied by a chip of the spreading code; then CWPT modulated signals are transmitted via transmitter antenna. At the receiver, the RF signals are converted to baseband signals; then CWPT is performed on the signal samples for multi-carrier demodulation. After that, the output signals are sent into channel equalizer. After despreading and combining the desired signal at different subcarriers, the soft decided complex values \{\hat{x}_k\} via QPSK symbol demapper, the \{\hat{x}_k\} assigned to the code bits \{x_k\} are fed into turbo decoder for iterative decoding, thus the detected source bits \{\hat{b}_k\} are achieved.

III. PERFORMANCE ANALYSIS

In this paper, the performance of CWP-MC-CDMA system with turbo coding is investigated in Rayleigh fading channel. At the transmitter, the complex wavelet packet function \(f_m(t)\) \((m=1,\ldots,M)\) is taken as the signature waveform, and the shifting orthogonality among complex wavelet packet function \(f_m(t)\) can be given as \(f_m(t), f_{m'}(t-\delta T)\rangle=\delta(m-m')\delta(t)\) [4], where superscript * represents complex conjugate and \(\delta\) is the Cronecker function. \(d_k(i)\) corresponding to the QPSK complex symbol denotes the data symbol of the \(k\)th user, where \(k=1,\ldots,K, K\) is the active user number; and \{\hat{d}_k\} are assumed to be independent, identically distributed \((i.i.d.)\) random variables taking value \{0,\pm 1/\sqrt{2}\} with equal probability, they are from the turbo encoding and QPSK mapping of transmitted information bits \{b_k\}. \(E_b\) is the mean energy of the transmitted bit. The symbol period \(T_s\) corresponds to the minimum orthogonal shifting defined in complex wavelet packet; \(C_s=\{c_k(m), m=1,\ldots,M\}\) is Walsh-Hadamard code, which represents the \(k\)th user spreading code, the length of code, i.e., processing gain is equal to the number of sub-carriers \(M\). The transmitted baseband signal of user \(k\) is written as:

\[
S_k(t) = \sum_{m=1}^{M} \sum_{i=0}^{2^{m-1}-1} \sqrt{2E_s} \cdot C_s(m) \cdot f_m(t-iT_s) \tag{1}
\]

In this paper, the frequency selective Rayleigh fading channel is considered. Suppose each modulated sub-carrier experience independent and flat fading, so the low-pass impulse response of \(m\)th sub-carrier channel for the \(k\)th user can be represented as:

\[
h_{k,m}(t) = \alpha_{k,m}(t) \exp[j\varphi_{k,m}(t)] \tag{2}
\]

where \(h_{k,m}\) is complex Gaussian random variable with zero mean and variance \(\sigma^2\) [6], the amplitude \(\{\alpha_{k,m}\}\) are \(i.i.d.\) Rayleigh variables with second-moment \(\sigma^2\), and the phase \(\{\varphi_{k,m}\}\) are \(i.i.d.\) uniform variables in the interval \([0,2\pi]\) for different \(k,m\). Considering downlink transmission, all user signals pass through the same channel, thus, for arbitrary \(m\), \(\alpha_{k,m}(t)=\alpha_{m}(t), \varphi_{k,m}(t)=\varphi_{m}(t)\). Suppose that all users receive the same power via power control. So at the receiver, after down converting to baseband, the received signal can be expressed by

\[
r(t) = \sum_{k=1}^{K} \sum_{i=1}^{M} \sum_{m=0}^{2^m-1} 2E_s/Md_k(i) \cdot c_k(m) \cdot h_{k,m}(t-iT_s) \cdot \exp(j\varphi_{k,m}(t)) + n(t) \tag{3}
\]

where \(n(t)\) is AWGN noise with double sided power spectrum density \(N_0/2\) and zero mean. We assume that the amplitude and phase remain constant during symbol period, i.e., \(\alpha_{k,m}(t)=\alpha_{m}(t), \varphi_{k,m}(t)=\varphi_{m}(t), h_{k,m}(t)=h_{m}(t)\exp(j\varphi_{m}(t))\). After passing through lowpass filter (LPF) and the complex wavelet packet matched filter in subchannel \(l\), the output signal at the \(nT_s\) sampling interval is

\[
y_l(u) = \sum_{k=1}^{K} \sum_{i=1}^{M} \sum_{m=0}^{2^m-1} 2E_s/Md_k(i) \cdot c_k(m) \cdot h_{m}(u)R_{l,m}^u((u-u)T_s) + n(u) \tag{4}
\]

where \(R_{l,m}^u(t) = \int_{-T}^{T} f_m(t-t) \cdot f^*_m(t) \cdot dt\), \(n(u) = \int_{-\infty}^{\infty} n(t) f_t(t-uT_s) \cdot dt\)

Without loss of generality, let user 1 be the desired user, then the decision variable for the \(u\)th data symbol of the user 1 is

\[
\hat{d}_u(u) = \sum_{l=1}^{M} c_l(l)A_l y_l(u) \tag{5}
\]

\[
= \sum_{l=1}^{M} \sum_{k=1}^{K} \sum_{i=1}^{M} \sum_{m=0}^{2^m-1} 2E_s/Md_k(i+u) \cdot c_k(m) \cdot h_{m}(u)R_{l,m}^u((u+u)T_s) + Z
\]

where \(Z = \sum_{l=1}^{M} c_l(l)A_l \int_{-\infty}^{\infty} n(t) f_t(t-uT_s) \cdot dt\)

\(A_l\) represents channel equalization gain for the subcarrier \(l\). Here, equal gain combining (EGC) is adopted, so \(A_l = \exp(-j\varphi_l)\). Thus, (5) can be changed as follows:

\[
\hat{d}_u(u) = \sum_{k=1}^{K} \sum_{l=1}^{M} \sum_{i=1}^{M} \sum_{m=0}^{2^m-1} 2E_s/M d_k(i+u) c_k(m) R_{l,m}^u((u+u)T_s) c_l(l) h_{m}(u) A_l
\]

\[
+ \sum_{k=1}^{K} \sum_{l=1}^{M} \sum_{i=1}^{M} \sum_{m=0}^{2^m-1} 2E_s/M d_k(u+u) c_k(m) R_{l,m}^u((u+u)T_s) c_l(l) h_{m}(u) A_l
\]

\[
= I_1 + I_2 + I_3 + D + Z \tag{6}
\]

where \(Z\) is Gaussian noise term with zero-mean and the variance \(\text{Var}(Z)=MN_0/2\). There are three types of interference contained in (6), (i) \(I_1\): interference from the same sub-channel \(l\) and the same user \(k=1\); (ii) \(I_2\): interference from the other sub-channels and the same user; (iii) \(I_3\): interference from the other users \(k\neq 1\). Considering
the cross-correlation functions of optimized complex wavelet packets [4] satisfy the equation \( \mathcal{R}^a_{m-n}(T)=\delta(m-n)\delta(j) \), then \( I_i=I_k=0 \), and 
\[ I^2 = \frac{2E_x}{M} \sum_{k=1}^{K} \sum_{i=1}^{M} d_i(u)c_i(l)c_i(l)a_i. \]

According to the previous assumption that \( \alpha_i \) is Rayleigh distributed with second-moment \( \sigma^2 \), as well as Central Limit Theorem (CLT) can be used to calculate the variance of \( I_i \) for large \( K \) and \( M \), so the variance can be written as:
\[ \text{Var}(I_i) = \frac{E_x}{M}[(K-1)(4-\pi)]\sigma^2/2 \quad (7) \]

where \( R_{4k} \) represents correlation between user \( k \) and user \( l \)'s spreading code. Owing to the superior orthogonality of Walsh-Hadamard codes, \( R_{4k}=0 \). So \( \text{Var}(I_1) \) can be rewritten as
\[ \text{Var}(I_1) = \frac{E_x}{M}[(K-1)(4-\pi)]\sigma^2/2 \quad (8) \]

For simplicity of analysis and calculation, channel coding (turbo coding) is not considered firstly, then \( \{d_k\} \) denote QPSK symbols from mapping of information bits \( \{b_k\} \) accordingly. Hence, we can calculate the probability of error conditioned on \( \{\alpha_i\}, i=1,2,...,M \) via (8) and variance of \( Z \) as follows:
\[ P(e|\alpha_i) = 0.5\text{erfc}\left(\frac{\sqrt{E[D]^2}2[\text{Var}(I_1)+\text{Var}(Z)]}{2}\right) \quad (9) \]

Let \( \alpha = \sum_{i=1}^{M} \alpha_i \), then \( D = \frac{2E_x}{M}d_i(u)\alpha \).

Considering \( M \) is large in practice, the Law of Large Number (LLN) can be employed to calculate \( \alpha \) approximately, i.e., \( \alpha \approx \frac{M\text{E}[\alpha_i]}{\sqrt{\pi}/2} \). Thus (9) is changed by \( P(e|\alpha, l=1,2,...,M) = P(e|\alpha) \), and the average bit error rate can be obtained via averaging \( P(e|\alpha) \) over \( \alpha \)
\[ P = \int_{0}^{\infty} 0.5\text{erfc}\left(\frac{\sqrt{E[D]^2}2[\text{Var}(I_1)+\text{Var}(Z)]}{2}\right) p(\alpha)\,d\alpha \]
\[ = 0.5\text{erfc}\left(\sqrt{\frac{M\sigma^2/2}{(K-1)(4-\pi)}}\right) \quad (10) \]

where \( p(\alpha) \) is the probability density function (PDF) of \( \alpha \), and \( \text{SNR}=E_x/N_0 \).

The equation (10) above is BER expression for uncoded CWP-MC-CDMA system. When using turbo codes as channel codes, the system BER expression need making corresponding changes. It is assumed that channel code rate \( R_c=m/n \), where \( m \) is the number of source bits and \( n \) is the number of resulting code bits at the output of the channel encoder. In the uncoded case, \( R_c = 1 \).
Using (9), the BER for coded-system without considering the decoding can be given by
\[ P = 0.5\text{erfc}\left(\sqrt{\frac{M\sigma^2/2}{(K-1)(4-\pi)}}\right) \quad (11) \]

After multiple iteration decoding, the turbo decoder can correct burst bits error and random bits error very effective, so the probability of bits error is much lower than the Eq.(11) above, thus the system BER performance is improved significantly.

In addition, data decision will be performed after turbo decoding, not after channel equalization and combining above. Refer to Fig.2 for specified principle descriptions, where the turbo encoder and decoder structure are shown in [5,7]. In this paper, the turbo encoder employs two 8 state and 1/2-rate identical recursive systematic convolutional (RSC) encoders (1, 1+D+D'/1+D+D') connected in parallel with an interleaver preceding the second RSC encoder, and the pseudorandom interleaver is selected as internal interleaver in the encoder. Both RSC encoders encode the information bits. The first encoder operates on the input bits in their original order, while the second encoder operates on the input bits as permuted by the turbo interleaver. Considering the coded bits block of each encoder consists of systematic bit part and parity bit part, and the systematic parts are the same in both coded bit streams, hence the systematic part is transmitted only once. The coded bits are not punctured, we choose the systematic bit from encoder1 as transmitted information bit, and add two parity bits, so the total code rate \( R_c \) of turbo encoder equals to 1/3, namely, one bit input, then three bits output. The turbo decoding is performed iteratively; the basic principle of iterative decoding is to feed forward/backward the decoder output (hard decisions and reliability information) to improve the next decoding. Refer to [7,5] for detailed decoding processes. By multiple iterative run, the performance can be significantly improved. Finally, the decision variable \( b_k \) for transmitted bit \( b_k \) is obtained. Considering the tradeoff between different decoding algorithm performance and realization complexity, the Max-Log-MAP algorithm is adopted.

IV. SIMULATION RESULTS

In this section, the performance of MC-CDMA system based on complex wavelet packet and turbo coding is simulated channel model A defined by ITU-R M.1225. The simulation results are shown in Fig.3 and Fig.4, respectively. In simulation, we assume that all users receive the same power and channel estimation is perfect. The orthogonal Walsh-Hadamard code is selected as spread code, and the processing gain equals to the number of subcarriers. The QPSK with Gray mapping is applied for turbo coded information bits, and EGC is exploited for channel equalization. The correlated parameters is set as follows: carrier frequency: \( f_c=2GHz \), the sampling rate: \( f_s=3.84 MHz \), the vehicle velocity: \( v=50km/h \), the bit rate per user: 384kb/s. Optimized complex wavelet packet with five-level of binary wavelet packet tree and four-level of binary wavelet packet tree [4] are considered in Fig.3 and 4, respectively. The parameters of turbo codes are defined as Section III.
Fig. 3 gives the average bit error probability as a function of $E_b/N_0$ under the condition that number of user $K=7$ and the number of subcarrier $M=16$. "DFT" and "CPxDFT" denote the conventional MC-CDMA system based on DFT without CP and with CP, respectively, where 'x' represents that x cyclic prefix are inserted in each multicarrier symbol (i.e. corresponding to one OFDM symbol). "CWP" denotes the CWP-MC-CDMA system. "TC1CWP" and "TC2CWP" denote turbo coded CWP-MC-CDMA system with 1-iteration and 2-iteration for decoding, respectively. From Fig. 3, the performance of CWP-MC-CDMA system outperforms that of MC-CDMA system based on DFT, and is close or superior to that of conventional MC-CDMA system with CP. Especially, turbo coded CWP-MC-CDMA system can improve the system performance significantly; it can increase the CWP-MC-CDMA system performance 3-5dB on the condition of the same bit error rate. To further comparison, we also give the average BER of different multi-carrier system against different $E_b/N_0$ in Fig.4, where $K=14$, $M=32$, and iterative times for turbo decoding are 1 and 2, respectively. From the simulation results indicated in Fig.4, we may conclude that turbo coded CWP-MC-CDMA system has best performance; and it can get lower BER than the other comparative system without coding, which accords with the theoretical analysis before. All these results show that turbo coded CWP-MC-CDMA system has strong robustness against various interference. Moreover, with the increase of sub-carrier numbers, the performance of the proposed system is improved further due to the frequency diversity gain from multi-carrier modulation. Conclusions

V. CONCLUSIONS

On the basis of analyzing the principle of multi-carrier CDMA technology, by employing the optimized complex wavelet packet as multi-carrier modulation and the turbo codes as channel coding, we propose a CWP-MC-CDMA system based on turbo coding, investigate the system downlink performance in Rayleigh fading channel, and give corresponding BER expression. The CWP-MC-CDMA system can overcome the loss of spectrum efficiency of conventional MC-CDMA due to inserting CP, and outperform MC-CDMA system based on DFT. Meanwhile, the performance of CWP-MC-CDMA system is also superior to that of MC-CDM with CP, thus the spectrum efficiency and system performance are increased further. Moreover, the system make full use of turbo codes' good ability to cope with the burst error of fading channel, as well as soft input and soft output information from multiple iterations decoding to make the date decide correctly. As a result, the system BER in fading channel is significantly decreased. Therefore, the application of turbo codes effectively improves system capacity against the channel fading and MAI.

REFERENCES

Fig. 1 Transmitter of Turbo coded CWP-MC-CDMA system

Fig. 2 Receiver of Turbo coded CWP-MC-CDMA system

Fig. 3 BER against SNR for different system

Fig. 4 BER against SNR for different system